

Instructions: Complete each of the following exercises for practice.

1. Evaluate the line integral $\int_{\partial R} P \, dx + Q \, dy$ via Green's Theorem.

(a) $P(x, y) = ye^x$, $Q(x, y) = 2e^x$;
 $R = [0, 3] \times [0, 4]$

(d) $P(x, y) = y^4$, $Q(x, y) = 2xy^3$;
 R : the ellipse $x^2 + 2y^2 \leq 2$

(b) $P(x, y) = x^2 + y^2$, $Q(x, y) = x^2 - y^2$;
 R : triangle with vertices $(0, 1)$, $(2, 1)$, and $(1, 0)$

(e) $P(x, y) = y^3$, $Q(x, y) = -x^3$;
 R : the ball $x^2 + y^2 \leq 4$

(c) $P(x, y) = y + e^{\sqrt{x}}$, $Q(x, y) = 2x + \cos(y^2)$;
 R : the region bounded by $y = x^2$ and $x = y^2$

(f) $P(x, y) = 1 - y^3$, $Q(x, y) = x^3 + \exp(y^2)$;
 R : the annulus $4 \leq x^2 + y^2 \leq 9$

2. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ via Green's Theorem.

(a) $\mathbf{F} = \langle y \cos(x) - xy \sin(x), xy + x \cos(x) \rangle$
 C : the triangle with vertices $(0, 0)$, $(0, 4)$, $(2, 0)$

(c) $\mathbf{F} = \langle y - \cos(y), x \sin(y) \rangle$
 C : the clockwise circle $(x - 3)^2 + (y + 4)^2 = 4$

(b) $\mathbf{F} = \langle e^{-x} + y^2, e^{-y} + x^2 \rangle$
 C : the arc of $y = \cos(x)$ from $(-\frac{\pi}{2}, 0)$ to $(\frac{\pi}{2}, 0)$
and then the segment connecting $(\frac{\pi}{2}, 0)$ to $(-\frac{\pi}{2}, 0)$

(d) $\mathbf{F} = \langle \sqrt{x^2 + 1}, \arctan(x) \rangle$
 C : the triangle with vertices $(0, 0)$, $(1, 1)$, $(0, 1)$